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# Adjoint bulk scalars and supersymmetric unification in the presence of extra dimensions

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## Abstract

There are several advantages of introducing adjoint superfields at intermediate energies around  $M = 10^{13}$  GeV. Such as (i) gauge couplings still unify (ii) neutrino masses and mixings are produced (iii) primordial lepton asymmetry can be produced. We point out that if adjoint scalars have bulk excitations along with gauge bosons whereas fermions and the doublet scalar live on boundary then  $N = 2$  supersymmetric beta functions  $\tilde{b}_i$  vanish. Thus even if extra dimensions open up at an intermediate scale  $\mu_0$  and all  $N = 2$  Yang-Mills fields as well as  $N = 2$  matter fields in the adjoint representation propagate in the bulk, still gauge couplings renormalize beyond  $\mu_0$  just like they did in 4-dimensions with adjoint scalars. Consequently unification is achieved in the presence of extra dimensions, mass scales are determined uniquely via Renormalization Group Equations(RGE) and unification scale remains high enough to suppress proton decay. This scenario can be falsified if we get signatures of extra dimensions at low energy.

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We know that the gauge group of Glashow-Weinberg-Salam(GSW) standard model is  $G \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ . Now suppose after compactifications string theory gives us a group  $G \times G$  and the scalars are only in fundamental representation of each component group. Then during the symmetry breaking  $G \times G \rightarrow G^{diag}$  to the diagonal  $G$  we obtain scalars in the adjoint representation. For example consider the symmetry breaking  $SU(3) \times SU(3) \rightarrow SU(3)^{diag}$ . Nine scalars of  $(3,3)$  representation transform after symmetry breaking as  $1 + 8$  of  $SU(3)^{diag}$ . The singlet can get a VEV for the symmetry to break and the adjoint octet is produced. Furthermore  $(1,3)$  or  $(3,1)$  representation become fundamental representation of  $SU(3)^{diag}$ .

It was noted by Bachas Fabre and Yanagida[1] that when we add the matter superfields  $(8,1,0)+(1,3,0)+(1,1,0)$  to Minimal Supersymmetric Standard Model(MSSM) spectrum at a scale of around  $10^{12-13}$  GeVs, the gauge couplings still unify but the unification scale is pushed up from  $10^{16}$  GeVs to the string scale of around  $10^{18}$  GeVs. This case becomes more interesting from the current experimental results on the neutrino masses as the fermionic partners of  $(1,3,0)$  and  $(1,1,0)$  scalars are capable of producing tree level diagrams giving Majorana masses to neutrinos[2]. This mass is of the order of  $m_Z^2/M$  where  $M$  is the mass of the adjoint fermions. Thus to get the mass of neutrino in the 1 eV range the scale  $M$  is required to be  $10^{13}$  GeV as suggested by the Renormalization Group(RG) analyses[3]. Note that the triplet  $T$  and the singlet  $S$  has the gauge invariant Yukawa couplings  $LH_2S$  and  $LH_2T$ . Following this line it can also been shown that out of equilibrium decay of either  $S$  or  $T$  could produce a tiny lepton asymmetry in the early universe which leads to leptogenesis[3].

Suppose  $\delta$  numbers of extra space-time dimensions open up at a scale  $\mu_0$  which is much below the Planck scale of  $10^{19}$  GeVs. In this case the running of the gauge couplings beyond  $\mu_0$  no longer remains logarithmic. Furthermore beyond  $\mu_0$  the Yang-Mills and matter superfields are arranged in  $N = 2$  multiplets. Thus  $N = 1$  beta functions  $b_i$  for the gauge couplings are to be supplemented by  $N = 2$  beta functions  $\tilde{b}_i$ . Thus *a priori* it is not at all guaranteed that the gauge couplings will preserve unification. In this context a very important work of Dines Dudas and Gherghetta(DDG)[4] shows that for MSSM the gauge couplings do not only unify

in 4-dimensions which is well-known but also unify in the presence of extra dimensions. This unification is independent of the scale  $\mu_0$  and the number of extra dimensions  $\delta$ . Indeed this is a remarkable property of the low energy particle content of MSSM. Following this paper a number of recent studies in this direction have been performed[6, 5, 7, 8]. Beyond the gauge structure of MSSM grand unification with intermediate left-right symmetry has also been considered[9].

Following this line we wish to ask what are the properties of MSSM enhanced by adjoint matter fields ? Does it preserve unification of gauge couplings in the presence of enlarged extra dimensions? Furthermore after unification of couplings can we uniquely predict right mass scale of extra states such that correct neutrino mass can be generated ? Clearly for us a new scale has been introduced which is  $\mu_0$ . In this paper we will analyze this case and find out a specific embedding of the superfields in higher dimensional space-time which will give rise to gauge coupling unification. Note that there exists a non-trivial change in the nature of gauge coupling evolution beyond  $\mu_0$  as the adjoint scalars remain in the bulk and hence the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  gauge bosons also live in the bulk. This is due to the string constraint that bulk fields transform under bulk gauge groups; this issue has been discussed by Carone[6] for example.

An  $N = 2$  theory consists of  $N = 2$  Yang-Mills fields as well as  $N = 2$  matter fields.  $N = 2$  Yang-Mills theory has  $N = 1$  Yang-Mills fields plus one Wess-Zumino multiplet in the adjoint representation of the Yang-Mills gauge group.  $N = 2$  matter consists of Wess-Zumino multiplets in  $R_i$  and  $\overline{R}_i$  representations. Thus for each additional adjoint matter representation below  $\mu_0$  that we are going to introduce we will have their Kaluza-Klein (KK) excitations leading effectively to a pair of adjoints above  $\mu_0$  to complete the  $N = 2$  hypermultiplet. This is because adjoint representations are self-conjugate. This is in the same spirit as of fermions where members of each pair above  $\mu_0$  has opposite charges. Thus vector-like fermion pairs above  $\mu_0$  not only complete  $N = 2$  hypermultiplet but also cancel anomaly. The  $N = 2$  beta function coefficients are expressed as[10]

$$\tilde{b}_i = 2[-C_2(G_i) + \sum_i T_i]. \quad (1)$$

Here  $C_2(G_i)$  is the quadratic casimir of the  $i$  th group  $G_i$ . If  $G_i$  is  $SU(N)$  then  $C_2(G_i) = N$ . There are  $\eta$  number of fermion generations experiencing extra dimensions and they contribute via the second term. Then we can explicitly isolate fermion contributions and write

$$\tilde{b}_i = 2[-C_2(G_i) + \sum_S T_S] + 4\eta. \quad (2)$$

Now the sum exclude fermion generations. To check DDG case let us combine doublet Higgs into a  $N = 2$  multiplet (in this case it contains two low energy  $N=1$  Higgs doublets<sup>2</sup>) in the bulk along with  $SU(2)$  gauge bosons. We immediately get  $\tilde{b}_2 = -3 + 4\eta$  as in Reference[4]. At this point suppose the adjoint scalars live in the bulk whereas the doublet scalar and fermions live on the boundary then we get<sup>3</sup>

$$\tilde{b}_i = 0. \quad (3)$$

Now we must examine the running of the gauge couplings beyond  $\mu_0$  scale.

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln\left[\frac{\Lambda}{\mu_0}\right] + \frac{\tilde{b}_i}{2\pi} \ln\left[\frac{\Lambda}{\mu_0}\right] - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[\left(\frac{\Lambda}{\mu_0}\right)^\delta - 1\right]. \quad (4)$$

Here  $b_i$  are  $N = 1$  coefficients including the adjoint scalars.  $\Lambda$  is the scale at which the couplings are being evaluated,  $\delta$  is the number of extra dimensions,  $X_\delta$  is given by,

$$X_\delta = \frac{2\pi^{\delta/2}}{\delta\Gamma(\delta/2)}, \quad (5)$$

and,  $\Gamma$  is the Euler gamma function. Thus we have implicitly assumed that  $\mu_0 > M$ , where  $M$  is the mass scale from which the adjoint scalars start contributing to RG analysis. We have

$$b_1 = 33/5, \quad b_2 = 3, \quad b_3 = 0. \quad (6)$$

Using Eqn. (3) the running equation between  $\mu_0$  and  $\Lambda$  in Eqn. (4) reduces to,

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln\left[\frac{\Lambda}{\mu_0}\right]. \quad (7)$$

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<sup>2</sup>Rather than combining two Higgs fields into an  $N = 2$  multiplet another possibility would be to augment each Higgs field to its own  $N = 2$  multiplets.

<sup>3</sup>To cross check, see Table[2] of Reference[7]:  $\Delta\tilde{b}_2=4$  for (1,3,0) and  $\Delta\tilde{b}_3=6$  for (8,1,0)

Now it is trivial to supplement Eqn. (7) with the running of the gauge couplings from  $m_Z$  to  $M$  using the MSSM beta functions  $\beta_i^{MSSM}$  and from  $M$  to  $\mu_0$  using  $b_i$ . Finally we obtain,

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(m_Z) - \frac{\beta_i^{MSSM}}{2\pi} \ln\left[\frac{M}{m_Z}\right] - \frac{b_i}{2\pi} \ln\left[\frac{\mu_0}{M}\right] - \frac{b_i}{2\pi} \ln\left[\frac{\Lambda}{\mu_0}\right]. \quad (8)$$

This equation can be simplified to

$$\alpha_i^{-1}(\Lambda) = \alpha_i^{-1}(m_Z) - \frac{\beta_i^{MSSM}}{2\pi} \ln\left[\frac{M}{m_Z}\right] - \frac{b_i}{2\pi} \ln\left[\frac{\Lambda}{M}\right]. \quad (9)$$

Thus as long as  $\mu_0 > M$  the scale  $\mu_0$  disappears from the renormalization group analysis. The coefficients  $\beta_i^{MSSM}$  are given by[11],

$$\beta_1^{MSSM} = 33/5, \quad \beta_2^{MSSM} = 1, \quad \beta_3^{MSSM} = -3. \quad (10)$$

Using the low energy experimental values of the three couplings it is very easy to solve equations (9) for unification scale  $M_X$ , intermediate scale  $M$  and unification coupling  $\alpha_X$ . We obtain one-loop result  $\alpha_X^{-1} = 20.45$ ,  $M_X = 10^{18}$  GeV and  $M = 10^{12.85}$  GeV which matches well with[12]. However above  $\mu_0$  we have  $N = 2$  supermultiplets thus it is sufficient to carry out one-loop RG analysis above  $\mu_0$ . Because in this case  $\mu_0 > M$  we obtain,

$$\mu_0 > 10^{12.85} \text{ GeV}. \quad (11)$$

Note as long as the condition (11) is satisfied, independent of the value of  $\mu_0$  and independent of the number of extra dimensions the unification scale  $M_X = 10^{18}$  remains invariant. Similarly independent of the scale  $\mu_0$  the intermediate scale is fixed by RGE to be  $M = 10^{12.85}$ . These are the main results of this paper. If we had power law running we would not have achieved a splitting of  $10^5$  GeVs between  $\mu_0$  and  $M_X$ .

Now the phenomenological motivation of this paper will be clear. It was shown recently [3] that a fermion triplet and a fermion singlet at around  $10^{13}$  GeV gives neutrino masses and mixings that explain solar and atmospheric neutrino oscillations. Furthermore the out of equilibrium decay of the triplet to  $H_2$  and  $L$  gives rise to a lepton asymmetry of the universe. We see that these nice features can be maintained in a similar unified model in the presence of extra dimensions.

Next we ask what if  $\mu_0 \ll M$  ? In this case we can consider bulk excitation of the doublet Higgses so using Eqn. (1) we get  $N = 2$  supersymmetric beta functions,

$$\tilde{b}_1 = 3/5 + 4\eta, \quad \tilde{b}_2 = -3 + 4\eta, \quad \tilde{b}_3 = -6 + 4\eta, \quad (12)$$

where  $\eta$  is the number of generations experiencing extra dimensions. Thus along with the MSSM coefficients we recover the scenario proposed by Dines Dudas and Gherghetta[4, 5] which gives unification close to the scale  $\mu_0$ . Variations of this scenario with minimal particle content can also be found[6]. There also exist detailed study of unification scale by introducing various extra multiplets[6, 7]. Note that in Table 13 of Reference [7] representations contained upto 75 of SU(5) are discussed. These representations actually include the adjoint fields of zero hypercharge that we are considering. However one extra state at a time was included in the RG analysis and thus highest unification scale was found for the introduction of (3,6,1/3) fermion at  $M_X = 1.3 \times 10^{14}$  GeV. We can also entertain the possibility that below the scale  $\mu_0$  two copies of (8,1,0)+(1,3,0)+(1,1,0) survives. Then  $N = 1$  beta functions are

$$b_1 = 33/5, \quad b_2 = 5, \quad b_3 = 3. \quad (13)$$

In this case we get  $M = 10^{15.42}$ ,  $M_X = 10^{18}$  and  $\alpha_X^{-1} = 20.45$ . Note that one of the main reasons on introducing (1,3,0)+(1,1,0) is to produce neutrino mass[2] via see-saw mechanism[13]. In the case of  $M = 10^{15.42}$  GeV we get that neutrinos can be at most as heavy as  $10^{-2}$  eV. Thus it will not be possible to address LSND results[16] where  $\Delta m_{e\mu}^2 \sim 1eV^2$  is required. They may also be too light for the purpose of atmospheric neutrino oscillation[17] where  $\Delta m^2 \sim 10^{-2}$  is the best fit region. Furthermore for neutrinos to become the hot component of dark matter[18] its mass needs to be in the 1 eV range.

Is it possible to have  $\mu_0 < M < M_X$  ? We have not found any such scenario with the choice of our fields that is MSSM supplemented by adjoint scalars. If more extra particles are added it may be possible to produce delicately balanced  $\tilde{b}$  and  $b$  coefficients to produce  $\mu_0 < M < M_X$ . This possibility is beyond the scope of present work because we are adding only adjoint superfields which are natural light remnants of string compactifications.

On proton decay one can take one of two viewpoints. Interesting ideas have been forwarded where compactifications is done on a  $Z_2$  orbifold where interactions that lead to proton decay vanish at orbifold fixed points. For example we can assign a discrete reflection symmetry depending on the extra dimensions under which the propagators mediating proton decay are odd whereas all fermions are even[4]. A more extended discrete symmetry depending on extra space-time dimensions has been also suggested recently in Reference[14] where matter fields have  $Z_2 \times Z'_2$  charge assignments which forbids proton decay. One can also consider gauging and breaking an additional U(1) symmetry on a distant brane[15]. These are higher dimensional solutions of proton decay problem. Otherwise in the worse case if these mechanisms are absent one must have the traditional solution of large  $M_X > 10^{16}$  GeV and get propagator suppression. In the second case the results obtained in this paper will be quite usefull in the presence of enlarged extra dimensions with  $\mu_0 > 10^{12.85}$  GeV.

In summary, radiative corrections in supersymmetric theories in the presence of extra dimensions come from two sources. Fields remaining on the boundary as well as fields propagating in the bulk give logerithmic corrections. These corrections are are of the form  $\frac{(b-\tilde{b})}{2\pi} \ln(\frac{\Lambda}{\mu_0})$  where  $b$  is  $N = 1$  beta functions. Fields propagating in the bulk form  $N = 2$  multiplets. We note that power law corrections to gauge couplings come exclusively from the fields propagating to the bulk. In the case where we have adjoint hypermultiplets propagating in the bulk whereas fermions as well as doublet Higgses staying on the boundary,  $N = 2$  beta functions vanish making the  $N = 2$  part finite. Now as power law corrections to gauge couplings are proportional to  $N = 2$  beta functions  $\tilde{b}$  they are absent. Thus in our example radiative corrections to gauge couplings maintain their logerithmic nature even in the presence of enlarged extra dimensions that is when  $\Lambda > \mu_0$  as displayed in Equation (8). Unification is delayed till  $M_X = 10^{18}$  GeV. This our main result which is independent of number of extra dimsions and the scale  $\mu_0$  as long as  $\mu_0 > 10^{12.85}$  GeV.

A theory can become  $N = 2$  finite in other ways too. For example if we add  $2N$  fundamental scalars to the theory,  $\tilde{b}$  will also vanish. However logerithmic corrections imparted by  $2N$  fundamental representations of matter above  $\mu_0$  will not give gauge coupling unification. This can be

easily understood by examining 4th term in the right hand side of Equation (8). Hence our example is unique where  $N = 2$  beta functions vanish and also gauge couplings unify in the presence of extra dimensions.

Another thing we must notice that when  $\mu_0 \ll M$  we have accelerated unification when doublet Higgs is in the bulk. Whereas if  $\mu_0 > M$  we have logarithmic unification. Thus the nature of evolution changes in a nontrivial way when  $\mu_0$  crosses  $M$ . Thus Equation (11) which is a result of RG analysis is a strong constraint on our scenario and it can also be used to falsify our scenario. We know that there are a number of studies which focus on experimental signatures of large extra dimensions at the TeV scale[19]. If we get positive signature of extra dimensions at low energy our scenario will be ruled out.

Finally as we have conventional running, we get many advantages of GUTs. The proton decay rate is suppressed because of large  $M_X$ . The scale  $M$  plays the role of the see-saw scale and we can get neutrino mass in the 1 eV region as suggested by neutrino oscillation experiments. Neutrino can also be a dark matter candidate in this case. The adjoint triplet and singlet can decay via Yukawa interactions and can lead to Leptogenesis. It will be an interesting idea to examine RGE of Yukawa couplings in this scenario and calculate fermion mass hierarchies.

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